# Lab 3 manual: Measurements and Uncertainties

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For PHYS 1111L and PHYS 2211L

(Lab outcomes: Introduction into measurements, uncertainties, and error propagation)

## Abstract

As a measurement is done, the obtained value contains the uncertainty. When used in equation, this uncertainty must be propagated to the result. The equations for this propagation are derived in this document. The procedure for ‘measurements and uncertainties’ lab is outlined. The procedure for measuring the systematic and statistical errors is provided.

## Introduction

The propagation of the uncertainties (normally called errors) and the ability to classify them as systematic and statistical are at the base of the theory of measurements. The presented derivations follow from the simple considerations and are useful in a multitude of fields beyond physics (think about which ones? You can include that in your introduction). Pay attention to the lab report formatting as more and more points will be subtracted for mistakes.

## Experimental Setup

A reminder, the experimental setup and should include the photo or a schematic of the setup (all relevant parts, such as scales, caliper, etc.) as well as the setup text description and the experimental procedure. The schematic can be very simple, such as shown in Figure 1. Briefly describe all the parts of your figure. Descriptions like Figure 1a or Figure 1 (left) can be used. Note the formatting of the figure reference and the caption. No colored text unless required by context.

In this experiment, the metal ball (Figure 1b) will be measured using other parts of the setup (yes, the description should be included)

Ruler

b)

a)

Figure : The experimental setup schematic. A ruler (a) is used to measure the object (b). Add missing components as needed. DON’T COPY FROM MANUAL!

### Procedure[[1]](#footnote-1)

We will use the scales, a metal ball, a metric ruler (never use imperial system in the lab!) and a caliper. Familiarize yourself with a caliper first before using it. The final result is the density of the ball. (Note: don’t do the %discrepancy unless the accepted value for the result is known. For example – in this lab, we don’t know the “true” value of the result for the metal ball, but we do know it for the aluminum ball, so this step depends on which ball is used.).

As we use the tools that are somewhat different from each other (i.e. the scales have different calibration and tuning, the balls are slightly different) and every person also tends to measure things a bit different as well, we need to assess both the systematic and the statistical errors. Note that in the future labs we will skip such rigorous measurements but remember that any error estimation that you will be making in the future can always be checked by a similar process.

First, we weigh the metal ball:

1. For estimating the statistical error of weighing, we need to reduce the contribution of different scales and different students. Thus:

One student will weigh all metal balls using the same scales. The results should be recorded by all groups. The stdev of these results provides you the systematic error on . That is, we assume that all balls should be identical, and the difference between them is a systematic error.

Hint: manufacturer precision is not always known. For the ruler, 0.5 mm can be used. Normally, half or quarter of smallest division can be used with caveat. Must clearly state that in the text.

1. Now pick one metal ball. At least 10 different students should weigh it using the same scales. Make sure that during the process you don’t speak your result but write it down silently and move the weights on the scales to hide your result. This is important! (WHY?). At the end, share the results with all the groups. The average of these results will be the used as , and the stdev – as the **statistical** error of the weighing process. (NOTE: sanitize hands and/or ball in the process as needed, make sure the ball is dry when weighing.)
2. One student will weigh the same ball using all different scales in the lab (minimum of 5). Share the results will all groups. The stdev of this data is the systematic error of the scales calibration.

After finishing the weighing process, each group will choose one ball and work with it only.

1. Measure the ball diameter using the ruler – yes, its not easy so be creative. Use the manufacturer precision as systematic error. Large error is expected. Share your results with all groups, and take the average as the radius and the stdev as the **statistical** error.
2. Measure the diameter of the ball using caliper. It is a precision measurement device so you can do a single measurement (suggestion: each group member should do a measurement and results should be compared. Ideally, they will match exactly, thus you can’t use the stdev here). When there is no detectable deviation in measurements, the **statistical** error is taken to be a half of the smallest division scale on the instrument – this is especially useful for the instruments with digital readouts when all digits are stable on the screen. (Hint for the future: if the readout value is unstable, record at least 10 values and use the average and stdev). The systematic error is the instrument precision provided by the manufacturer. **Don’t share this result, its only for your group**.

In the theory section, provide the equations that you are going to use with explanation of symbols in them (for volume of the sphere and for density). You can skip theory section and include all that into the analysis section instead. Don’t include equations for the average and stdev as they are assumed to be common knowledge (unless you used non-standard equation for the stdev – include it and provide explanation as to why).

**Show the equations for the error propagation and their derivation** (PHYS 2211L only). The key derivation steps must be shown.

For PHYS 1111L – **only** **show the error propagation equations** that you will be using as you will be using simplified results from later sections in this manual.

Remember to number the equations. Use equation editor in Word or your text processing software, the equations typed in text like: s^2-√(23/45)^3 and all similar will not be accepted.

If you find the volume separately from density – show two equations. If combine first into the single formula – one. Any mistakes in this part will carry the **heaviest** penalty so make sure you are doing this correctly!

You should find 2 separate density values, one for using the ruler and one for using the caliper. Compare them to each other (and to the accepted value if known) within errors. Can do % discrepancy as well here if accepted value is known.

Instructions on how to handle all different errors that we have measured:

2211L: combine all systematic errors into a single systematic as:

and all statistical ones into the single statistical error value by the adding them in quadrature as well. Propagate both systematic and statistical errors **separately** and give results with 2 errors. Reminder: show propagation equation derivation using partial derivatives.

**1111L only**: combine all systematic and statistical errors together into a single error using . Use simplified error equations. Extra credit for showing derivation using partial derivatives.

Extra credit for all: find confidence interval (say 95%) on ball mass and explain what it means for weighting other balls from the lab kits not used in the lab.

## Experimental Data

The results of the measurements are in Table 1. Think about how to organize this table. The error can be listed in a different column or as value ± error unit. Don’t include all data values here, only relevant means, stdev etc. All data values may be included in the Appendix.

Table : Experimental data.

|  |  |
| --- | --- |
| Measured quantity | Measured value |
| Radius (average) |  |
| Mass (average) |  |
|  |  |
|  |  |

Provide description about each entry as needed, don’t just leave the table without text in this section. Organize well so that data takes less space but everything needed is included.

## Theory

Consider some equation (that is completely made up here) that described the dependence of quantity x on measured quantities m1, m2, c and e – shown in equation 1. Let’s say that in this example m=Average (m1, m2) since both are very close – like each metal ball has its own mass but we take m to be the same for all with some error. And same error is used for m. Since c, e, m1 and m2 are measured, they have an associated uncertainty that will contribute to the .

But before we write out the equation for the , we need to derive it first.

### General error analysis/propagation (not for PHYS 1111L)[[2]](#footnote-2)

General theory: When you have a formula of the form , and either x or y (or both) have uncertainties (errors) associated with them, then the error propagation equation is the full differential of , meaning that you take a partial derivative of *f* in each variable that has an associated error. The details and explanation of what is a partial derivative will be shown during the lab meeting on error analysis.

For example, say that but z is either known perfectly somehow, or the error in z is orders of magnitude smaller relative to value of z, and we can skip it for the illustration of this example. Then the error of the function will be:

where denote the partial derivatives and the and are the corresponding uncertainties.

Note that both the error in x and y are assumed to be independent and uncorrelated from each other and are added in quadrature just as two vectors would be. The actual reason is different, but this will help you remember it.

Also, a partial derivative of with respect to x and y is used. These are used with functions that depend on more than one variable. It is easy evaluate a partial derivative: in the first term with x, take the regular derivative in x but treat y as if it were a constant.

### Simplified error propagation equations[[3]](#footnote-3)

Now that you have seen the full derivation, note that the error analysis in the lab report should be kept simple yet used everywhere it’s applicable. The simpler equations can be derived from the equation 1 for common cases using simple formulas of forms like , without trig or log functions, etc. Table 2 shows how to do error analysis for simple formulas:

Table : Simplified error propagation equations.

|  |  |
| --- | --- |
| Formula form, | Simplified error analysis rules to be used |
|  |  |
|  |  |
|  |  |

However, for the formulas involving more complex function like trigs and exponentials, you will have to use the full differential formula (not in PHYS 1111L). Remember that errors are always positive. And they always increase no matter if you add, multiply or divide variables with each other. Exceptions are powers, say, a square root makes the error to be reduced in half, but the ratio of error to its value still increases!

(NOTE: if you divide or multiply the variable by a **constant**, the associated error gets divided or multiplied by the same constant. Addition or subtraction doesn’t affect the error).

### Error estimation method (supplemental):

There is a short-hand version of error analysis for a simple function that contains a term with a trig, log or similar slow varying function (will not work for exponential functions!) that you can’t process using simplified equations. Do not use in the lab reports unless you are in PHYS 1111L – but you can use this for quick error estimation and for checking if you calculated errors right).

Say , with and . Then the quick error estimate will be:

Note that in the first term, you use the values that give you the largest result (for example, for cos(a) you would use the angle minus error instead). And second term should be the smallest result that can be obtained by applying all errors.

If you want to convince yourself that this works, compare this with the result from the differentiation using equation (2). Note that radians must be used for the for the calculations!

## Analysis

Example 1. What about equation 1? Now we can find the uncertainty in x by propagating the errors for the m, c and e:

The equation 3 has the term for m three times because it’s the number of occurrences of the variable in the equation. However, if the power of the variable is different, it should be treated separately when using these simplifies error propagation rules.

You only need to show the important parts of the error analysis calculations (very brief), but include the error with any measured number and value unless the initial error was much smaller than the value measured so that it could be taken as ‘perfect’ (e.g. time in computer measurements it counted typically in seconds., but the uncertainty is in milliseconds so it can be omitted).

Example 2 (not for PHYS 1111L). Given , we need to propagate the error and find the . Then the error on d will be:

Note that the partial derivative in x is done using the chain rule in x only. **in radians!**

(Numbering rule: you don’t have to number equations that you will not refer to later in the report. But numbering them is not an error)

## Conclusion

This article describes the format for the lab report as well as the initial error analysis and propagation. The procedure of the measurements and uncertainties lab experiment is provided with details and instruction. Remember to include references and appendices as needed.

1. In your report, DON’T COPY this or any other section. This document is a lab manual and, in many parts, will be written in the style that is not appropriate for the report. I will consider copying any text from manual as error and lack of section even if you reference it, or even as plagiarism. Must be in your words, describing what you did in the lab. Might be different from this description as lab activities may not always follow the description provided exactly. [↑](#footnote-ref-1)
2. The lab manual will indicate sections/subsections/paragraphs that are not required for PHYS 1111L students. Such sections will count as extra credit only for PHYS 1111L if a student were to correctly complete the work in that section. Sections labeled as ‘extra credit’ are not required for all students but will count as extra credit for all courses. [↑](#footnote-ref-2)
3. If there is no indication that the section is extra credit or is not for some group of students, it’s for **everyone**! [↑](#footnote-ref-3)